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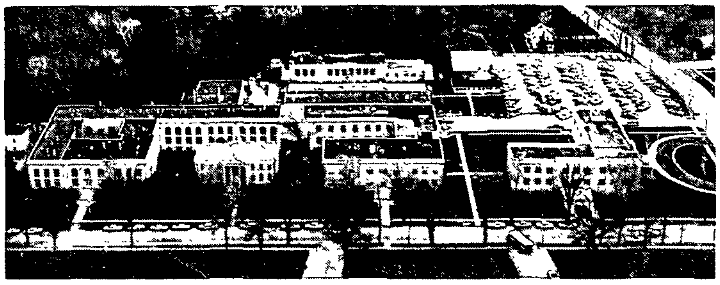

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ESTIMATING POISSON RATIOS IN PAPER USING ULTRASONIC TECHNIQUES

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Estimating Poisson ratios in paper using ultrasonic techniques

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ABSTRACT

Two simple methods for estimating Poisson ratios in paper are described. Equations relating the in-plane Poisson ratios of an orthotropic planar material to the in-plane ultrasonic velocities have been derived and experimentally tested. Ultrasonic velocities were measured on kraft linerboard using two experimental systems. The calculated Poisson ratios were compared with values measured using a biaxial tester developed at The Institute of Paper Chemistry. Agreement between the measured and calculated ratios was quite good. Both the ultrasonic and biaxial methods, however, are very sensitive to small changes in the experimental data.

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Introduction

The mechanical behavior of paper and board is important in almost all converting and end use applications. In particular, resistance to failure is essential in the various papermaking, converting, or printing operations. Fundamental studies of failure in paper or board, or containers made from these materials, require a knowledge of the elastic constants. In recent studies [e.g., Reference (1)], the material is assumed to behave as an orthotropic planar material where the elastic constants of interest are those in the plane. These in-plane elastic constants are: the two Young's moduli, E_x and E_y, corresponding to the machine and cross-machine directions, respectively; the shear modulus, G_{xy}; and the two Poisson ratios, ν_{xy} and ν_{yx}. The Poisson ratio ν_{xy} is the ratio of the lateral contraction in the x-direction to the axial extension in the y-direction when the material is stressed uniaxially in the y-direction. The Poisson ratio ν_{yx} is defined in a similar way. Only four of the elastic constants in the plane are independent. If one of the Poisson ratios is known the other can be calculated, according to the theory of elasticity for an orthotropic material, from

$$\nu_{xy}/E_y = \nu_{yx}/E_x. \tag{1}$$

Of the in-plane elastic constants, the Poisson ratios are perhaps the most difficult to measure via mechanical tests. In paper stressed uniaxially, the lateral strain is very small and difficult to measure directly, especially if the paper tends to warp out of the plane during straining. Such problems are described by Jones (2), who demonstrated that paper behaves as a two dimensional orthotropic material.

Biaxial straining of the paper specimen, where stresses are simultaneously applied in both the machine and cross-machine directions, is perhaps one of the best methods for measuring Poisson ratios. The procedure is quite complicated, however, and requires a special apparatus and a considerable length of time (hours) to perform. This paper is concerned with simpler methods of determining the in-plane Poisson ratios. Two techniques are described in which either three or four ultrasonic velocities are measured and used to calculate the in-plane Poisson ratios. Values so obtained agree favorably with values determined using the biaxial testing method. The measurements and calculations can be carried out in a short period of time.

This work has been part of a broader study concerned with elastic wave propagation in paper. The theoretical and experimental aspects involved in treating paper as a three dimensional platelike orthotropic material are described elsewhere (3, 4).

Theory

Craver and Taylor (5, 6) were the first to apply sonic velocity measurements to paper. Assuming that paper was a two-dimensional anisotropic, homogeneous, elastic media, they developed the equations relating sonic velocities to the in-plane elastic constants E_x, E_y, and G_{xy}. They did not, however, develop analytical expressions for Poisson ratios in anisotropic paper in terms of the sonic velocities. Since they were primarily interested in the Young's moduli in the plane, and the Poisson effects were assumed to be small, they either ignored the Poisson ratios altogether (and defined a "sonic modulus"), or they assumed that as far as these constants are concerned, the paper behaves as if it were isotropic, i.e., $\nu_{xy} = \nu_{yx}$.

In the development which follows, paper is assumed to be a homogeneous elastic, orthotropic media subjected to a plane stress state (stresses in the thickness direction are zero). For such a system, the following generalized Hooke's Law expression applies (7),

$$\varepsilon_i = \Sigma_j S_{ij} \sigma_j, \tag{2}$$

where i, j = 1, 2, or 6. The ε_i, σ_j, and S_{ij} are the strains, stresses (dyne/cm^2), and elastic compliances (cm^2/dyne), respectively. In this discussion we assume that the 1 (or x) direction corresponds to the machine direction in the paper, 2 (or y) to the cross-machine direction, and 6 to the shear in the x-y plane. For an orthotropic material, $S_{16} = S_{26} = S_{62} = S_{61} = 0$. Conversely, the stresses may be expressed in terms of the strains as

$$\sigma_i = \Sigma_j C_{ij} \varepsilon_j, \tag{3}$$

where the C_{ij} are the elastic stiffness coefficients (dynes/cm^2).*

Suppose now that an ultrasonic wave is propagating in the plane of the two dimensional system. The equations of motion for the disturbances are:

$$\begin{aligned} \frac{\partial \sigma_1}{\partial x} + \frac{\partial \sigma_6}{\partial y} &= \rho \frac{\partial^2 u_1}{\partial t^2}, \\ \frac{\partial \sigma_6}{\partial x} + \frac{\partial \sigma_2}{\partial y} &= \rho \frac{\partial^2 u_2}{\partial t^2}, \end{aligned} \tag{4}$$

where u_i is a small displacement in the i-direction, and ρ is the mass density of the medium. The strains are defined in terms of the displacements:

*The values of the stiffness constants defined for the planar case differ from those defined for the three dimensional case, whereas the compliances are the same in either two or three dimensions.

$$\varepsilon_1 = \frac{\partial u_1}{\partial x}$$

$$\varepsilon_2 = \frac{\partial u_2}{\partial y} \tag{5}$$

$$\varepsilon_6 = \left(\frac{\partial u_2}{\partial x} + \frac{\partial u_1}{\partial y}\right).$$

If Equations (3) and (5) are inserted into Equation (4), the equations of motion are, in terms of the displacements and the stiffness coefficients,

$$C_{11} \frac{\partial^2 u_1}{\partial x^2} + C_{12} \frac{\partial^2 u_2}{\partial x \partial y} + C_{66} \left(\frac{\partial^2 u_2}{\partial x \partial y} + \frac{\partial^2 u_1}{\partial y^2}\right) = \rho \frac{\partial^2 u_1}{\partial t^2}$$

$$C_{66} \left(\frac{\partial^2 u_2}{\partial x^2} + \frac{\partial^2 u_1}{\partial x \partial y}\right) + C_{12} \frac{\partial^2 u_1}{\partial y \partial x} + C_{22} \frac{\partial^2 u_2}{\partial y^2} = \rho \frac{\partial^2 u_2}{\partial t^2}. \tag{6}$$

For longitudinal waves propagating in the principal directions, two solutions are:

$$u_1 = u_{10} \exp[i(k_1 x - \omega t)], \; u_2 = 0;$$

$$u_2 = u_{20} \exp[i(k_2 x - \omega t)], \; u_1 = 0, \tag{7}$$

where ω is the angular frequency of the wave, k_i is the wave number, and u_{10} and u_{20} are amplitudes. Upon substituting Equations (7) into (6),

$$C_{11} = \rho \, c_x^{\,2} \; (= \omega^2/k_1^{\,2}),$$

$$C_{22} = \rho \, c_y^{\,2} \; (= \omega^2/k_2^{\,2}). \tag{8}$$

In these expressions, c_x and c_y are the longitudinal wave velocities in the machine and cross-machine directions, respectively. For a transverse wave, where the particle displacement is perpendicular to the direction of propagation, e.g., $u_1 = u_{10} \exp[i(k_2 y - \omega t)]$ and $u_2 = 0$, we obtain

$$C_{66} = \rho \, c_s^{\,2}, \tag{9}$$

where c_s is the velocity of the shear wave.

The matrices $\underline{S}_{ij}$ and $\underline{C}_{ij}$ in Equations (2) and (3) are inverses, making it possible to express the elastic stiffnesses in terms of the compliances. The latter, in turn, can be expressed in terms of the engineering constants of the system [e.g., Reference (7)],

$$
\begin{aligned}
S_{11} &= E_1^{-1} \quad (= E_x^{-1}) \\
S_{22} &= E_2^{-1} \quad (= E_y^{-1}) \\
S_{12} &= -\nu_{12}/E_2 \quad (= -\nu_{xy}/E_y) \\
S_{21} &= -\nu_{21}/E_1 \quad (= -\nu_{yx}/E_x) \\
S_{66} &= G_{66}^{-1} \quad (= G^{-1}).
\end{aligned}
\tag{10}
$$

Since $\underline{S}_{12} = \underline{S}_{21}$, Equations (8) and (10) reduce to

$$
\begin{aligned}
E_x &= \rho\, c_x^2\, (1 - \nu_{xy}\nu_{yx}), \\
E_y &= \rho\, c_y^2\, (1 - \nu_{xy}\nu_{yx}), \\
G &= \rho\, c_s^2.
\end{aligned}
\tag{11}
$$

These equations apply to wave propagation in the orthotropic plane stress state. Equations (11) were recently found to be the low frequency approximations in a more general theory which considers paper as a three dimensional orthotropic platelike material (3). At high frequencies, determined largely by the caliper and elastic modulus in the z-direction, the Equations (11) do not apply. For heavy linerboard samples, the planar assumption is valid only up to approximately 100 kHz. At higher frequencies, the out-of-plane elastic constants must be included.

Equations (11) were obtained by assuming wave solutions propagating along the principal directions in the paper, i.e., the machine or cross-machine directions. In the more general case, one may wish to obtain a solution for the case of a wave propagating at some arbitrary angle in the plane of the sheet. This can be done by assuming solutions of the form

$$u_1 = u_{10}\exp[i(k_x x + k_y y - \omega t)]$$
$$u_2 = u_{20}\exp[i(k_x x + k_y y - \omega t)]. \tag{12}$$

Equations (6) are now coupled, and must be solved simultaneously. In general, the solutions will no longer be pure longitudinal or transverse modes, that is, the particle displacements will no longer be parallel or perpendicular to the direction of propagation.

A particular case of interest here is that in which the wave propagates at an angle of 45° from the machine (or x) direction. In this case we let $k_x = k_y = k/(2)^{1/2}$, leading to the result (after some manipulation):

$$\rho\ (\omega/k)^2 = \rho\ c_{45}^2 = (1/4)(C_{11}+C_{22}+2C_{66}) \pm (1/4)[(C_{11}-C_{22})^2 + 4(C_{12}+C_{66})^2]^{1/2} \tag{13}$$

In this expression, using the negative sign, c_{45} is the velocity of a quasi-transverse wave propagated at 45° from the machine direction. Since ω and k are linearly related, the phase velocity ($= \omega/k$) and the group velocity ($= d\omega/dk$) are equal. Since the constants C_{11}, C_{22}, and C_{66} can be evaluated in terms of measurable velocities [Equations (8) and (9)], and c_{45} can also be measured, the stiffness constant C_{12} can be determined from Equation (13). From the relationship between the stiffness and compliance matrices, and using Equation (10), it can be shown that (for the orthotropic planar case)

$$C_{12} = \nu_{12}C_{11}. \tag{14}$$

By combining Equations (8), (9), (13), and (14), and solving for ν_{12} (or ν_{xy}), we obtain

$$\nu_{xy} = -\frac{1}{B} + \frac{1}{B}\left(1 + \frac{4B^2}{A^2} - \frac{2B^2}{A} - \frac{2B^2}{AR} - \frac{4B}{A} + \frac{B^2}{R} + \frac{B}{R} + B\right)^{1/2} \tag{15}$$

where $R = (c_x/c_y)^2$, $B = (c_x/c_s)^2$, and $A = (c_x/c_{45})^2$. Thus by measuring the four velocities, c_x, c_y, c_s, and c_{45}, we can determine ν_{xy}. Note that the expression is independent of the mass density.

It is also possible to deduce expressions which give the elastic constants at some angle, ϕ, measured from the machine direction. This can be done by a rotation operation on the stiffness or compliance matrices, without giving any consideration to wave propagation. Of particular interest in this discussion is the rotation transformation of the shear modulus, G. At some arbitrary angle ϕ the shear modulus may be written [e.g., Reference (7)], $G_\phi^{-1} = 4\cos^2\phi \sin^2\phi$ $(E_x^{-1} + E_y^{-1} + 2\,\nu_{xy}E_y^{-1}) + (\cos^2\phi - \sin^2\phi)^2\, G^{-1}$.

When $\phi = 45°$:

$$G_{45}^{-1} = E_x^{-1} + E_y^{-1}\,(1 + 2\nu_{xy}). \tag{16}$$

If it is assumed that G_{45} is given by ρc_{45}^2, Equations (11) and (16) give

$$\nu_{xy} = -\frac{1}{A} + \frac{1}{A}\left(1 + \frac{A^2}{R} - \frac{A}{R} - A\right)^{1/2}, \tag{17}$$

where A and R are defined as above. In this approach, it is possible to estimate a value for ν_{xy} by measuring only three velocities, c_x, c_y, and c_{45}. The value of ν_{yx} can then be found from $\nu_{yx} = R\,\nu_{xy}$. As before, the mass density does not appear in the expression.

Equation (17) is only an approximation, because G_{45} will not exactly equal $\rho\, c_{45}^2$. Equation (15) reduces to Equation (17) if B is replaced by A, that is, if c_s is replaced by c_{45}. For those papers studied to date, however, the differences between Equations (15) and (17) are small compared to the usual variation in specimens and the experimental error in measuring the ultrasonic velocities.

Results and discussion

The measured ultrasonic velocities are presented in Table I. Each entry is an average of two or more (up to ten) measurements on each specimen. The number in parentheses following each entry is the standard deviation in the averaged values. The latter may be attributed to both experimental procedure and local variations in the linerboard itself. Measurements using the IPC system were made on only one specimen of each of the three linerboard samples. Note that for these linerboard samples, c_{45} is typically less than c_s. This is expected from orthotropic elasticity theory.

[Table I here]

The Poisson ratios calculated from the measured velocities are given in Table II and compared with the values determined using the biaxial tester. The values ν_{xy} and ν_{yx} were computed using Equations (16) and (1). The values ν_{xy}^* were calculated according to Equation (15) using all four measured velocities. This equation should give the "true" value of ν_{xy}, assuming the four measured quantities are accurate. In the present case, ν_{xy} and ν_{xy}^* agree within about 6%.

[Table II here]

In the biaxial tests ν_{xy} and ν_{yx} are determined independently, and their ratio should also equal R. The fact that it doesn't suggests some experimental difficulty. Notice also that the variability in ν_{xy} and ν_{yx} from specimen to specimen is substantial.

In Table III the results for each sample of linerboard are averaged, and the three methods of estimating Poisson's ratios are compared. Assuming that the biaxial tester yields the 'correct' value, the numbers in parentheses give the percent differences of the value immediately above, compared to the biaxial results. In most cases the agreement is better than 30% although in some instances the differences are larger.

The values of Poisson's ratios calculated from the ultrasonic velocities are quite sensitive to the variations in the velocities. For example, a 5% error in measuring $\underline{c}_{\underline{x}}$ for specimen 69-2 results in a 4% error in the calculated $\nu_{\underline{xy}}$. A 5% error in either $\underline{c}_{\underline{y}}$ or $\underline{c}_{45}$, (not both), however, causes about a 25% difference in $\nu_{\underline{xy}}$. A 5% error in both $\underline{c}_{\underline{y}}$ and $\underline{c}_{45}$, in the same direction, produces only a 3% error. The success of the ultrasonic method is clearly quite dependent upon the ability to make accurate measurements of velocity. On the other hand, specimen variations are difficult to account for, and comparing Poisson ratios determined biaxially and acoustically may not be expected to yield perfect agreement. The acoustical measurements involve a number of measurements along a number of paths in each of three directions in the plane of the sheet. The biaxial method involves the measurement of stresses 'averaged' over the width of the specimen, but a strain measured between two points (pins) on the surface of the paper. Certainly more experimental verification of the acoustic method must be sought, together with improved measurements of Poisson's ratios using mechanical methods.

The methods described should be useful for estimating Poisson's ratios in the plane of the sheet. Since only three or four relatively simple measurements of velocity are required, the procedure is far less tedious than any of the corresponding mechanical methods. We have found that for paper grades displaying less anisotropy than linerboard, and which are more uniform in properties in the plane of the sheet, the acoustically determined Poisson's ratios from specimen to specimen vary little.

[Table III here]

Experimental

The acoustical measurements were made using two different techniques. A Dynamic Modulus Tester PPM-5R, manufactured by the H. M. Morgan Co., was used in conjunction with an oscilloscope in one of the techniques. In the other method, only the point contact transducers from the Morgan instrument were used, together with a pulse generator, power amplifier, oscilloscope, and electronic counter. This is referred to as the IPC system.

In either method, two piezoelectric transducers contact the specimen. One acts as a transmitter, creating a mechanical disturbance when it is excited electrically, while the second detects the mechanical disturbance and puts out an electrical signal. The transducers deflect along a single axis, so it is possible to propagate both longitudinal or transverse (shear) waves by proper orientation of the two transducers. For longitudinal waves the displacement of a particle is in the direction of propagation of the wave, whereas for a transverse wave the particle displacement is perpendicular to the propagation direction.

Longitudinal or shear wave velocities are determined by measuring the transit time for the mechanical disturbance to propagate between the two transducers at a given separation distance. The velocity is the ratio of separation distance to transit time. In order to eliminate time delays in the cables or associated electronics, the following procedure was adopted in all measurements. Transit times were measured for a number of separation distances, typically between 3 and 25 cm, at one centimeter intervals. The data, plotted as separation distance versus transit time, form a straight line whose slope is the velocity. The slope was determined by performing a linear regression on the data.

When using the Morgan instrument, transit times were read directly from the oscilloscope connected to the front output jack on the Morgan instrument. The threshold adjustment on the Morgan instrument was adjusted according to the instruction manual for the device. Transit times were read to the nearest microsecond.

The IPC system used for measuring ultrasonic velocities is more sophisticated than the Morgan instrument, in that the transmitting transducer is excited by a burst of sine waves, rather than a single triangular pulse, and the transit times can be measured very accurately with an electronic counter. The apparatus will be described in detail elsewhere (4), and only a brief description is presented here. A pulse generated by an Interstate F74 function generator and amplified by an ENI 240L power amplifier, drives the sending transducer. A square pulse, coincident with the generator output, is used to trigger the main base of a Hewlett-Packard 1740A oscilloscope, which coincidentally starts a Hewlett-Packard 5300B/5308A time interval counter. The signal received at the second transducer is amplified by a Panametrics 5050AE-160B preamplifier, and displayed on the oscilloscope. The instant of triggering of the second time base can be controlled by the operator, so that it is possible to trigger at any desired peak or crossover point in the burst of sine waves. If automatic triggering is employed, the system functions like a very accurate Morgan instrument. Coincident with the triggering of the delayed time base is a square wave output pulse which stops the time interval counter. The counter gives a continuous display, averaged over a selectable number of periods, to the nearest nanosecond.

The concept of biaxial testing is quite straightforward. From Equations (2) and (10), the strains ε_x and ε_y may be written in terms of the stresses:

$$\varepsilon_x = (\sigma_x/E_x) - (\nu_{xy}\sigma_y/E_y)$$

and

$$\varepsilon_y = (\sigma_y/E_y) - (\nu_{yx}\sigma_x/E_x).$$

Thus, if E_x and E_y are known, and ε_x, σ_x, and σ_y are measured simultaneously, ν_{xy} can be calculated from the upper equation. In like manner, if ε_y, σ_y, and σ_x are measured simultaneously, ν_{yx} can be calculated. In practice, E_x and E_y can be found from uniaxial studies.

For the biaxial studies, the specimens are cut into a cruciform shape, and a series of slits are cut into and parallel to the four arms of the cross, similar to those described by Mönch and Galster (8). The cruciform shape and the slits produce a relatively uniform stress field within the central region of the specimen when it is stressed biaxially. In the biaxial tester, all four arms of the specimen are clamped for biaxial tests, but either pair of clamps may be opened so that uniaxial tests may be performed. Strains are measured within the region of uniform stress by attaching two pins to the surface of the paper specimen. The pins, in turn, are mounted to an A-frame strain gage such that, as the specimen elongates, the arms of the A-frame separate. Strains can only be measured in one direction at a time. In use, the specimen is first stressed biaxially for one load-unload cycle, taking care not to enter the plastic regime of the stress-strain curve. The experiment is then repeated, measuring the strain in the machine direction while correspondingly measuring the stresses in both the machine and cross-machine directions. The clamps in the cross-machine direction are then released, and a uniaxial stress in the machine direction is applied and the strain measured. From this the modulus E_x may be calculated. The A-frame strain gage is then disconnected from the specimen and rotated so that strains in the cross-machine direction may be measured. The specimen is then stressed uniaxially in the cross-machine direction so that E_y is determined. Finally, the specimen is stressed biaxially once again, so that the strain in the cross-machine direction is measured simultaneously with the stresses in both directions. Both ν_{yx} and ν_{xy} can then be calculated. The anisotropy ratio, R, can be obtained by taking the ratio of the principal moduli, E_x/E_y, or by taking the ratio of ν_{yx}/ν_{xy}.

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I. Summary of experimental results

Sample	Ultrasonic velocities[a] (modified Morgan instrument), mm/μsec				Ultrasonic velocities[b] (using IPC system), mm/μsec		
	C_x	C_y	C_s	C_{45}	C_x	C_y	C_{45}
69-1	3.66(0.04)	2.13(0.12)	--	1.58(0.04)			
69-2	*3.36(0.11)	*1.98(0.05)	--	*1.59(0.06)	3.70(0.07)	2.20(0.01)	1.62(0.01)
69-3	3.62(0.05)	2.07(0.06)	--	1.59(0.03)			
Average	3.55	2.06		1.59	3.7	2.2	1.62
42-1	3.43(0.09)	2.09(0.05)	1.70(0.02)	1.62(0.05)			
42-2	*3.51(0.05)	*2.12(0.05)	1.69(0.01)	*1.60(0.08)	3.71(0.16)	2.26(0.01)	1.63(0.00)
42-3	3.49(0.04)	2.16(0.03)	1.76(0.02)	1.67(0.01)			
Average	3.48	2.12	1.72	1.63	3.71	2.26	1.63
26-1	3.54(0.14)	2.27(0.03)	1.81(0.03)	1.76(0.02)			
26-2	*3.63(0.09)	*2.22(0.05)	1.79(0.02)	*1.69(0.05)	3.75(0.06)	2.33(0.04)	1.71(0.04)
26-3	3.69(0.04)	2.18(0.06)	1.79(0.03)	1.69(0.08)			
Average	3.62	2.22	1.80	1.71	3.75	2.33	1.71

Values in parentheses are standard deviations.

[a]Each reported value for C_x or C_y is an average of three measurements, except those values marked *.
Each reported value for C_s or C_{45} is an average of four measurements, except those values marked *.

[b]Each reported value is an average of two measurements.

*Each reported value is an average of ten measurements.

II. Summary of calculated results

Sample	Modified Morgan instrument					IPC system				Biaxial tester			
	A	*R*	ν_{xy}*	ν_{xy}[a]	ν_{yx}[b]	*A*	*R*	ν_{xy}[a]	ν_{yx}[b]	*R*	ν_{xy}	ν_{yx}	ν_{yx}/ν_{xy}
69-1	5.37	2.95		0.166	0.490					3.46	0.145	0.552	3.81
69-2	4.47	2.88		0.086	0.248	5.22	2.83	0.170	0.481	3.16	0.160	0.424	2.65
69-3	5.18	3.06		0.135	0.413					3.45	0.149	0.565	3.79
Average		2.96		0.129	0.384					3.36	0.151	0.514	3.40
42-1	4.48	2.69	0.124	0.116	0.312					2.93	0.163	0.387	2.38
42-2	4.81	2.74	0.153	0.145	0.397	5.18	2.69	0.187	0.503	2.57	0.220	0.395	1.80
42-3	4.37	2.61	0.124	0.116	0.303					2.75	0.189	0.342	1.81
Average		2.68	0.133	0.126	0.337					2.75	0.191	0.375	1.96
26-1	4.04	2.43	0.108	0.104	0.253					--	--	--	--
26-2	4.61	2.67	0.142	0.134	0.358	4.81	2.59	0.168	0.435	2.96	0.150	0.339	2.26
26-3	4.77	2.87	0.131	0.122	0.350					2.77	0.234	0.418	1.79
Average		2.66	0.129	0.120	0.320					2.87	0.192	0.379	1.97

*Calculated using Equation (15).
[a]Calculated using Equation (17).
[b]Calculated from Equations (1) and (17).

III. Comparison of results

	Modified Morgan				IPC system			Biaxial tester		
	R	ν_{xy} *	ν_{xy} [a]	ν_{yx} [b]	R	ν_{xy} [a]	ν_{yx} [b]	R	ν_{xy}	ν_{yx}
69# Linerboard	2.96 (-12)		0.129 (-15)	0.384 (-25)	2.83 (-16)	0.170 (+13)	0.481 (-6)	3.36	0.151	0.514
42# Linerboard	2.68 (-3)	0.133 (-30)	0.126 (-34)	0.337 (-10)	2.69 (-2)	0.187 (-2)	0.503 (+34)	2.75	0.191	0.375
26# Linerboard	2.66 (-7)	0.129 (-33)	0.120 (-38)	0.320 (-16)	2.59 (-10)	0.168 (-12)	0.435 (+15)	2.87	0.192	0.379

Numbers in parentheses ( ) are percent differences from average biaxial values.
*Calculated using Equation (15).
[a]Calculated using Equation (17).
[b]Calculated using Equations (1) and (17).